Metric Directional Derivatives of Multifunctions and its Application to Local Approximation

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Abstract

We introduce metric divided differences and metric directional derivatives for multivariate set-valued functions, extending our previous results from the univariate setting. The presented framework includes notions of metric differentiability and metric (uniform and non-uniform) Gateaux differentiability, together with several fundamental properties.

Using these metric directional derivatives, we define local metric linear approximants for set-valued functions and investigate their structural and ap- proximation properties. In particular, we show that metrically differentiable multifunctions admit first-order local approximations with error estimates in the Hausdorff metric. For uniformly metrically Gateaux differentiable multi-functions, the approximation error is shown to be of order $o(|x - x_0|)$ and higher-order estimates are obtained under metric α-differentiability assumptions.